Optical conductivity of Bose droplets in quenched disorder at $T = 0$: the effects of structure factor.

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The optical response of a system of two-dimensional Bose condensate droplets with an internal charge structure interacting with impurities is considered within the mode-coupling theory. It is found, that the glass transition in the system of droplets occurs at much stronger disorder as compared to the case of stuctureless point particles. The optical conductivity of the system is found to exhibit a complex oscillating structure, intimately related to the charge distribution within the droplets. The application of the model to the problem of the far IR optical conductivity of doped cuprates $La_{2-x}Sr_xCuO_4$ ($x < 0.26$) is discussed.

Keywords: Superfluid-glass transition, memory function, Bose droplet

I. INTRODUCTION

In a recent paper[1], a simple model of the superfluid-glass transition in the system of 2D bosons at zero temperature have been presented. The exact analytic solution of the self-consistent equation for the memory function permitted to retrace the quantum phase transition of the Bose condensate from superfluid to the insulating (Bose-glass) regime, as the dimensionless parameter, measuring the strength of the short-range random potential of impurities (pinning centers) reaches some critical value.

In the present article we extend this model to consider the behaviour of the disordered system of Bose droplets with some internal charge structure, interacting with random impurities. The droplet may extend over a few unit cells on a crystalline layer and is a region of a quasi-free motion of a Bose particle. It may be regarded as a potential well, where the boson is confined, and the radius of the droplet corresponds to the relevant localization length. The droplet acts as a large heavy Bose quasi-particle (like a large polaron). To our mind, a promising physical example of the droplets are the local islands of superconducting phase of HTSC materials above T_c (see e.g. Ref.[2]), which existence is sometimes associated with charged stripes[3]. Our approach is based on the mode-coupling theory of diffusion and localization[4], that makes it possible to derive the memory function of the system, given the bare correlation functions and the parameters of the random potential.

The main qualitative result of our analysis is that the internal real-space structure of the droplet strongly affects the electromagnetic response of the system and gives rise to the novel features in optical conductivity spectra as compared to the case of structureless point Bose particles. In the next section we formulate the model and in Section 3 show the results. In Section 4 the applications of the model to the experimental data on microwave optical response of doped cuprates are discussed. We believe, that the collective density modes in the system of the Bose droplets may give rise to the features in the optical conductivity spectra, that are usually ascribed to the dynamics of the charged stripes.

II. THE MODEL

We consider the two-dimensional (2D) system of heavy Bose particles at $T = 0$, randomly distributed on a background with real static dielectric constant ε. Each particle is a droplet, carrying a charge, spread over the droplet with the density $\rho(\mathbf{r})$. The interaction between the droplets is taken into account within the random phase approximation, assuming, that the system is dilute enough and the droplets do not overlap.

Our goal is to study the optical response of the system in presence of the random impurity potential. The main quantity of interest in this problem is the memory function $M(k, z) = M' + i M''$, that makes it possible to write down the optical conductivity of the system in a general form:

$$\sigma'(z) = \frac{\omega_p^2 \, M''}{(z + M')^2 + M''^2} \qquad (1)$$

where ω_p is the plasma frequency, z – the complex (Laplace) frequency. As usual, the k-dependence is omitted in eq.(1), so that the memory function evaluated at $k = 0$ is of interest. Depending on the analytic properties of $M(z)$, eq.(1) yields the coherent optical response of a clean system, or describes the diffusive charge dynamics in a dirty localized phase. The common practical method of exploring the extended-localized phase transition in disordered system is provided by the mode-coupling theory[4,5], that yields the self-consistent equation for the memory function:

$$M(z) = i\,\gamma + \frac{1}{d\,n\,m} \int\limits_0^\infty \Phi\left(z + M(z); k\right) \langle |U(k)|^2 \rangle k^2 d\mathbf{k}, \qquad (2)$$

where γ is a bare relaxation constant, Φ is the density-density relaxation kernel[5], $U(k)$ is the Fourier component of the impurity potential, d, n, m are the dimensionality

of the system, the concentration and mass of the constituent charge carriers, respectively. The angular brackets $\langle \ldots \rangle$ stand for the average over the configurations of impurities. In the case of random configurations this reduces merely to the factor n_i, the concentration of impurities. It may be seen, that in absence of impurities $M = i\gamma$ and the optical conductivity, eq.(1), reduces to the simple Drude form. The approach, based on eqs.(1,2) have been applied to a number of model systems, both classical[6] and quantum Fermi[4] and Bose[5] fluids.

Denoting the random impurity potential as $U(r) = U_0\varphi(r)$, where $\varphi(r)$ is dimensionless, the properly scaled equation (2) for the memory function $M(0,z)$ of 2D Bose liquid at $T = 0$ K takes the form[7]:

$$M = \frac{8\Lambda}{\pi} \int\limits_0^\infty \frac{(z+M)\,\varphi(k)^2\,k^{\nu+3}}{(1+k^{2+\nu})\,(k^4 + k^{2-\nu} - z(z+M))}\,dk,$$
(3)

where $\nu = 1$ for Coulomb interparticle interaction, $V(r) \sim 1/r$, and $\nu = 2$ for logarithmic ("2D Coulomb") interaction, $V(r) \sim \log(r)$. This latter type of interaction is typical of the topological extended structures, such as the vortices in the type II superconductors. The ground-state properties of the 2D Bose gas with logarithmic interaction have been considered in Ref.[8]. All quantities in eq.(3) are dimensionless, the units of momentum and energy being:

$$p_0 = \left(\frac{8\pi\hbar m^* n_{2D} Q^2}{\varepsilon}\right)^{\frac{1}{2+\nu}}, \quad \varepsilon_0 = \frac{p_0^2}{2m^*}, \quad (4)$$

respectively. Here m^* is the effective mass of the droplet, Q – the charge, n_{2D} – their concentration and ε is the real static dielectric constant of the medium. The properties of the memory function are governed by a dimensionless parameter

$$\Lambda = \frac{\pi}{2}\frac{n_i}{n_{2D}}\left(\frac{U_0}{\varepsilon_0}\right)^2 = \frac{n_i m}{4n_{2D}^2}\left(\frac{U_0}{\hbar Q}\right)^2, \quad (5)$$

where n_i is the concentration of impurities.

Taking into account the internal structure of the droplet, the impurity potential $\varphi(r)$ should be averaged with its charge density $\rho(r)$. Hence, the Fourier transform $\varphi(k)$ is multiplied by the form-factor $\rho(k)$ of the droplet. Thus, new expression

$$\widetilde{\varphi}(k)^2 = \varphi(k)^2 S(k) \quad (6)$$

is to be put into eq.(3) instead of $\varphi(k)^2$, where $S(k) = |\rho_k|^2$ is hereafter called the structure factor. The form-factor of an isotropic 2D droplet is calculated using the formula:

$$\rho_k = 2\pi \int\limits_0^\infty \rho(r)J_0(kr)\,r\,dr, \quad (7)$$

where J is the Bessel function of the first kind.

III. RESULTS

A. Superfluid-glass transition

The model under consideration allows full analytic treatment[1] for the system of point 2D logarithmic droplets ($S(k) = 1$, $\nu = 2$) interacting with pinning centers – the random impurities with the short-range potential $\varphi(k) = 1$. The electromagnetic properties of such a model share much similarity with those of pinned charge density waves (CDW)[9]. The basic qualitative results, that apply to a broad range of superfluid-glass transition models[5,10], are as follow.

Until Λ is small, the low-frequency expansion of $M(z)$ is:

$$M \simeq m_1 z + \ldots, \quad (8)$$

($m_1 = const.$), and the optical conductivity (1) is singular: $\sigma' = \frac{\omega_p^2}{1+m_1}\delta(\omega)$, where $z = \omega + 0i$. Thus, the system is superfluid at small disorder. However, as the full spectral weight $\int_0^\infty \sigma'(\omega)d\omega$ is constant, the reduction of the $\delta(\omega)$-response by a factor $1 + m_1$ as compared to the case of free Bose-condensate entails the emergence of the spectral features at $\omega > 0$. The spectral weight of the δ-peak at zero frequency drops with increasing Λ and it vanishes at a critical value $\Lambda = \Lambda^*$. At this point, the memory function acquires the singularity, called the non-ergodicity pole, and in leading order in z:

$$M \simeq \frac{m_0}{z} + \ldots. \quad (9)$$

In this regime, the optical conductivity tends to zero in the limit of small z, i.e. the system is in insulating (glassy) phase. The onset of the localization corresponds to the condition: $m_0 \to 0$. Substituting this into eqs.(3,6,9) with z tending to zero, one obtains the following equation for Λ^*:

$$\frac{8\Lambda^*}{\pi} \int\limits_0^\infty \frac{\varphi(k)^2 S(k)}{(1+k^{2+\nu})^2} k^{2\nu+1} dk = 1. \quad (10)$$

The solutions of eq.(10) for point particles ($S = 1$) are summarized in Table I. In particular, for the model of 2D bosons with logarithmic interaction ($\nu = 2$) in short-range impurity potential ($\varphi = \delta(r)$) we recover the condition $\Lambda^* = 1$, obtained in Ref.[1]. Since Λ^* varies depending on the model parameters, it is natural to make use of a renormalized invariant constant Λ/Λ^*.

For droplets with a non-trivial structure factor, the integral in eq.(10) can be solved numerically. Whatsoever complicated were the structure factor $S(k)$, it is anyway obvious, that in this case Λ^* is always greater, than the corresponding value for point particles, since $S(k) < 1$ for non-zero k. In other words, the droplets cannot be localized as easily as the point particles. If the density n_i of impurities is high, this is an odd conclusion, since
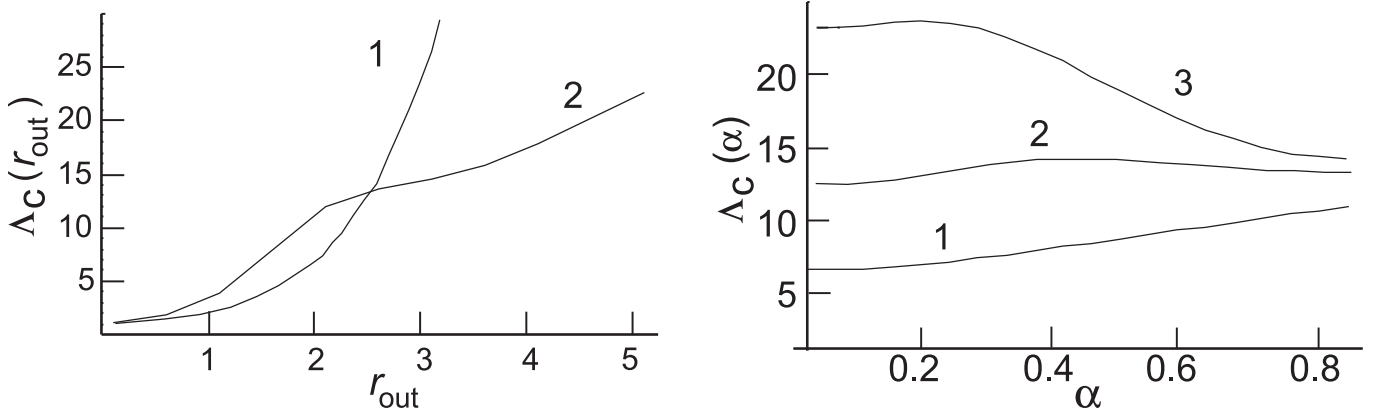
FIG. 1: Left panel: The dependence of the critical disorder parameter Λ^*, eq.(10), on the outer radius of the droplet for some inner-to-outer radius ratio α: $1 - \alpha = 0.01$, $2 - \alpha = 0.99$. Right panel: The dependence $\Lambda^*(\alpha)$ for different r_{out}: $1 - r_{out} = 2.0$, $2 - r_{out} = 2.5$, $3 - r_{out} = 3.0$.

TABLE I: Λ^* for point Bose particles ($S(k) = 1$) derived from eq.(10) for different impurity potentials φ and interparticle interactions ($\nu = 1$ – Coulomb, $\nu = 2$ – logarithmic).

$\varphi(r)$	$\delta(r)$		$1/r$	
ν	1	2	1	2
Λ^*	$\dfrac{9\sqrt{3}}{16}$	1	$\dfrac{9\sqrt{3}}{16}$	$\dfrac{\pi}{2}$

it seems likely that a large droplet gets stuck on a rugged potential landscape whereas a point particle may easier elude the obstacles. On the other hand, given a small number of deep potential wells, it is physically clear, that the point particle is trapped first, while a large droplet is bound less effectively, in accord with the above conclusion. Hence, making use of the terminology adopted in CDW theory[9], we can say, that the present model corresponds the regime of strong pinning (n_i low, U_0 large).

To be specific, we consider the droplets, that have the form of the uniformly charged rings with inner and outer radii denoted r_{in}, r_{out}, respectively. The charge density is: $\rho(r) = (\theta(r - r_{in}) - \theta(r_{out} - r))/S$, where $S = \pi(r_{out}^2 - r_{in}^2)$ is the area of the ring, whence the form-factor (7) is:

$$\rho_k = \frac{2}{1 - \alpha^2}\left(\frac{J_1(k\,r_{out}) - \alpha J_1(\alpha\,k r_{out})}{k\,r_{out}}\right), \qquad (11)$$

where $\alpha = r_{in}/r_{out}$. The droplet is a disk for $\alpha = 0$, and a thin ring for α close to unity. We show at Fig.1 (left panel) the dependence $\Lambda^*(r_{out})$ for some values of α as derived from eq.(10) for Coulomb droplets ($\nu = 1$) in the $\delta(r)$ impurity potential (other models, listed in Table I, yield very similar results). As expected, the critical level Λ^* of the disorder, at which the glass transition takes place, increases rapidly with the radius of the ring. It is also instructive to consider the dependence of Λ^* on the thickness α of the ring, its outer radius being fixed (Fig.1, right panel). Noteworthy is the non-trivial variation of
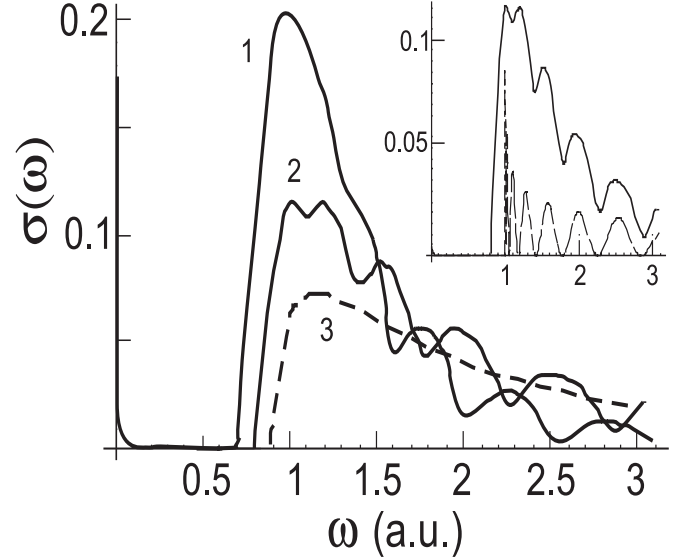

FIG. 2: The optical conductivity of logarithmic ($\nu = 2$) droplets with radius $r_{out} = 15$ and different ring thickness parameter α for $\Lambda/\Lambda^* = 0.1$: $1 - \alpha = 0$, $2 - \alpha = 0.9$, 3 – the spectrum of point Bose particles[1]. Inset: the curve 2 together with the properly scaled structure factor (dashed line) in rhs. of eq.(14).

$\Lambda^*(\alpha)$ curves with r_{out}. Two competing factors, the ring charge density and its spatial extension, affect the coupling with impurity. As α increases from zero, Λ^* begins to grow since the charge moves on average to a greater distance from impurity and couples weaker with it, albeit the charge density increases. However, this reasoning may become wrong for large droplets, where the character of $\Lambda^*(\alpha)$ dependence changes qualitatively. The behaviour of the curve 3 at right panel of Fig.1 means, that the non-ergodicity transition for thin rings (large α) of large enough radius may occur at lower Λ^*, than for disks, i.e. these rings are more readily trapped by

impurities.

B. Optical conductivity

Now consider the effects of the structure factor (11) on the optical conductivity spectra (1). Some typical examples of $\sigma(\omega)$ spectrum are shown at Figs.2,3. These results are obtained numerically from eqs.(1,3,6,11) given $\Lambda/\Lambda^* < 1$ (superfluid regime). The spectra display a broad asymmetric feature, related to the collective (plasmon-like) density modes[1,5]. As compared with the corresponding spectra of point particles with the same Λ/Λ^* ratio, that display a single smooth feature (dashed curves at Figs.2,3), the spectra of the droplets are slightly shifted to lower frequencies and exhibit a peculiar oscillating structure. The oscillations become more pronounced with increasing radius of the droplets and (or) inner-to-outer radius ratio α.

The origin of the oscillations can be understood as follows. The main contribution to the integral in eq.(3) comes from the vicinity of the wave numbers $k_i > 0$ ($i = 1, 2$):

$$k_i^4 + k_i^{2-\nu} - \operatorname{Re}\zeta = 0, \quad \zeta = z(z + M(z)), \qquad (12)$$

where the integrand has the peaked structure. In particular, as far as $\Lambda \ll \Lambda^*$, $|M(z)|$ is small and as a crude zero-order approximation, we may put $M = 0$ in rhs. of eq.(3), so that Im ζ is infinitesimal and the peaks are very sharp. Then, the imaginary part of eq.(3) takes the form:

$$M'' = \omega \frac{8\Lambda}{\pi} \int_0^\infty \sum_i R_i(k)\, S(k)\, \delta(k - k_i)\, dk, \qquad (13)$$

where $R_i(k)$ denotes the regular part of the integrand near $k = k_i$ and the structure factor is not included in R_i. The spectral dependence of the memory function and the optical conductivity (1) are thereby governed by the strongly oscillating expression $S(k_i(\omega))$. E.g., for $\nu = 2$ eqs.(12,13) yield

$$M'' = 2\Lambda \frac{\sqrt{\omega^2 - 1}}{\omega} S\left((\omega^2 - 1)^{1/4}\right), \quad \omega > 1. \qquad (14)$$

It may be seen at the inset of Fig.2, that the rhs. of eq.(14) reproduces correctly the oscillations of the optical conductivity spectrum.

As the amount of the disorder Λ/Λ^* increases, the oscillations of optical conductivity become less pronounced. The point is that the imaginary part M'' of the memory function and Imζ increase with Λ so that the δ-peaks in eq.(13) acquire a sizeable half-width Δk. Thus, the contributions of the structure factor $S(k)$ add up from a range of wave numbers $(k_i(\omega) - \Delta k, k_i(\omega) + \Delta k)$ and partially cancel each other.

We have investigated also more complex axially symmetric charge distributions. All of them reduce to a discrete or continuous series of concentric rings, considered above. The resulting optical spectra are found to display a system of sharp and flat peaks of the same origin as those depicted in Figs.2,3.

IV. DISCUSSION

The mode-coupling theory of the 2D weakly disordered Bose condensate have been first proposed for the interpretation of the optical response of doped HTSC oxides in Ref.[10]. Despite a number of model assumptions, the results obtained therein have been found to compare well with the properly scaled optical spectra of $YBa_2Cu_3O_{7-\delta}$.

To our opinion, the approach considered in Ref.[10] and in the present work deserves attention in connexion with the far-IR features in optical conductivity spectra of underdoped HTSC's, that are presumably related to the stripe excitations[11] or Wigner crystal dynamics[12]. In particular, the in-plane optical conductivity of $La_{2-x}Sr_xCuO_4$ ($0 < x < 0.26$) is found to display strong resonant structure around ~ 100 cm^{-1}, that consists of a main peak and a series of less pronounced peaks at higher frequencies[11]. The authors interpret these features as the signature of the optical response of charged stripes, extending over many ($\sim 10^2$) cells in the Cu-O plane.

These results are only a part of a great body of findings[13] obtained with different techniques (neutron scattering[14], microwave spectroscopy[12,15], scanning tunnel microscopy[16], etc.), that give convincing evidence of the existence and an important role of a real-space inhomogeneity in the physics of doped cuprates and related systems. The electronic inhomogeneity, considered mainly in the form of density waves or stripes, suggests the percolative nature of the transition between the insulating and superconducting regions of the $T_c - x$ phase diagram of HTSC's[17,18]. The latter implies the existence of the nanoscale islands of superconducting phase above T_c, and it was proposed, that "the superconductivity arises predominantly in the charged stripes (or domains)"[3].

The present model of Bose droplets may be relevant for the description of electromagnetic response of such a system of local condensate patches. The droplets emerge on Cu-O planes of layered HTSC's upon doping as the local bosons with charge Q and effective mass m^*, delocalized within a region of a certain spatial extension, that depends on the width of the potential wells created by doped impurities. The droplet may be either rounded or "stripe"-shaped – in the latter case the droplets should be randomly oriented so as to restore the isotropy of the system. At finite frequencies the optical spectrum of the system consists of a broad asymmetric feature (the plasmon wing) due to the collective plasmon-like excitations of the droplet arrays. The latter is similar to the pinned CDW optical response, and it was recently shown[1], that the optical conductivity spectrum of the 2D bosons with logarithmic interaction in random short-range potential
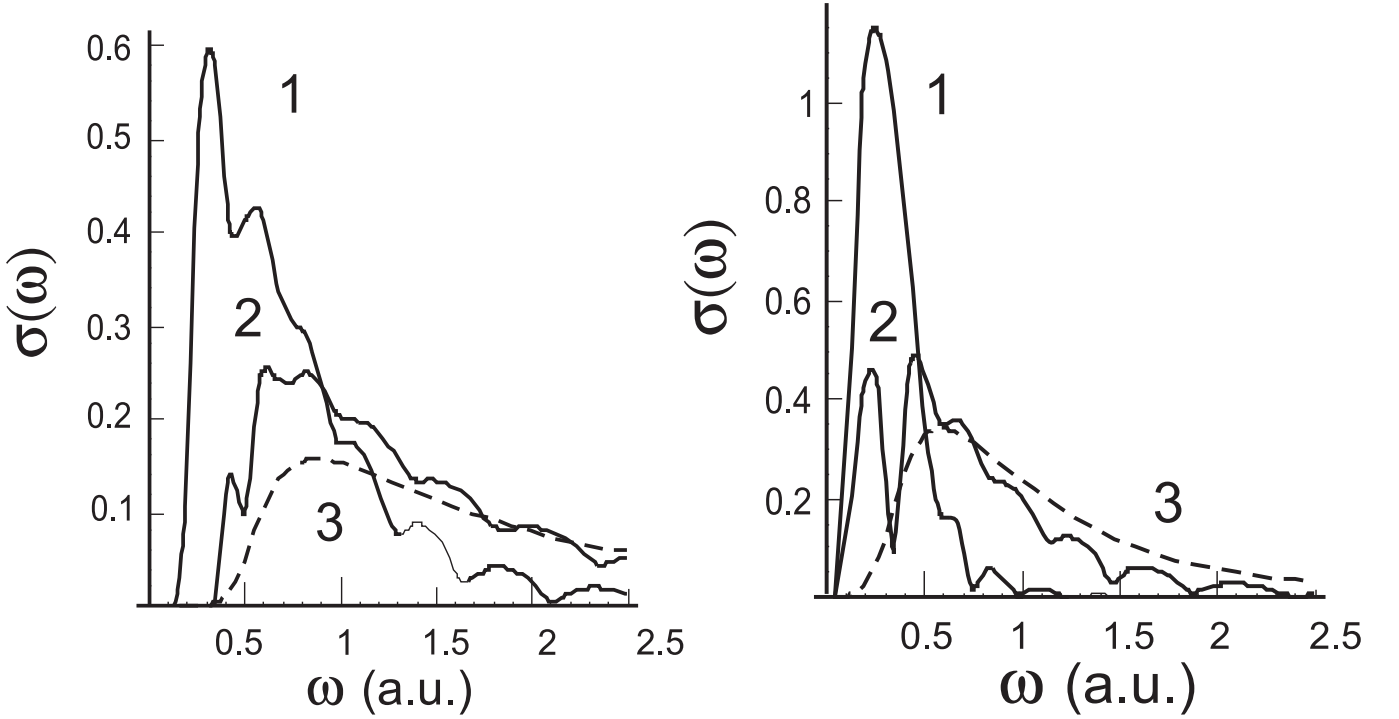
FIG. 3: Optical conductivity of Coulomb droplets with $r_{out} = 15$ and different real-space structure in superfluid regime, $\Lambda/\Lambda^* = 0.25$. $1 - \alpha = 0$ (circles), $2 - \alpha = 0.9$ (rings), 3 – point particles (no structure). Left panel: short-range impurities ($\varphi = \delta(r)$). Right panel: Coulomb impurities ($\varphi = 1/r$).

($\nu = 2$, $\varphi(r) = \delta(r)$) exhibits just the same frequency dependence as the CDW in strong pinning regime[9]. However, as far as the Bose liquid remain confined within the droplets, its real-space density is essentially nonuniform, and the effects of a non-trivial structure factor are important. These manifest themselves in the strong frequency modulation of the plasmon wing (Figs.2,3), governed by the structure factor (eq.14).

We point out that the spectra at Fig.3 bear a good deal of resemblance with microwave optical conductivity of lightly doped $La_{2-x}Sr_xCuO_4$ at $x \sim 0.1$, that is supposedly dominated by charged stripes excitations[11]. The in-plane concentration of the droplets is assumed to depend on the doping concentration x. Thus, $x = 0.1$ corresponds to $n_{2D} = 6.25\,10^{-12}$ cm^{-2} (the area of the CuO_4 cluster being 16 Å^2). Taking $Q = 2e$, $m^* = 3000\,m_e$ and the static dielectric constant $\varepsilon = 40^{12}$ (e and m_e being the electron charge and mass, respectively), we obtain the unit of energy (eq.4) $\varepsilon_0 = 204.25$ cm^{-1} and the unit of length $\ell = 2\pi\hbar/p_0 \simeq 15$ Å. Hence, the oscillating structure of the spectra at Fig.3 falls to the energy ~ 100 cm^{-1} and the droplet size is ~ 225 Å, or about 40 CuO_4 cells. These estimates are in agreement with the energy of charged stripes excitations and the mean stripe size, reported in[11].

Assuming, that each impurity creates a droplet on CuO_2 plane and $n_{2D} \sim n_i$, it follows from eq.(5), that $\Lambda \sim 1/x$. However, it is not Λ itself but the ratio Λ/Λ^* that governs the shape of the optical conductivity spectra, where Λ^* (eq.10) depends on the structure of the droplet and in particular, on its radius. A rigorous estimate of the droplet radius could be obtained by minimization of free energy and is beyond the scope of the present work. However, since we regard the droplet as a region of a coherent motion of the boson, it is plausible, that its radius reduces with increasing doping concentration x, when the system becomes inhomogeneous at smaller scale, whereby Λ^* decreases with x. As Λ^* varies strongly with the droplet radius r_{out} (see Fig.2), this implies, that the ratio Λ/Λ^* may grow with x, although Λ and Λ^* decrease.

At Fig.4 we show a series of spectra, that simulates the variation of the microwave optical conductivity of $La_{2-x}Sr_xCuO_4$[11] with doping. The left panel corresponds to the disks ($\alpha = 0$) in short-range impurity potential, $\varphi(k) = 1$. At early stage of doping (Fig.4A) there exists a small number of large droplets with $r_{out} = 30.5$ and $\Lambda_A/\Lambda_A^* = 0.05$. In these conditions the spectrum consists of an almost flat background and a single distinct feature, similarly to the optical conductivity of LSCO at $x \sim 0.07$. At Fig.4B it evolves into the wing with a well-pronounced structure, as discussed in previous section. This spectrum corresponds to $x \sim 0.15$ and is obtained with $\Lambda^B : \Lambda^A \simeq 0.07 : 0.15$ and reduced droplet radius $r_{out} = 15$, that provides $\Lambda_B/\Lambda_B^* = 0.2$. The spectrum at Fig.4C corresponds to $x \sim 0.26$ and is obtained given $\Lambda_C : \Lambda_B \simeq 0.15 : 0.26$ and still smaller droplets with $r_{out} = 8$, so that $\Lambda_C/\Lambda_C^* = 0.5$. It is seen,
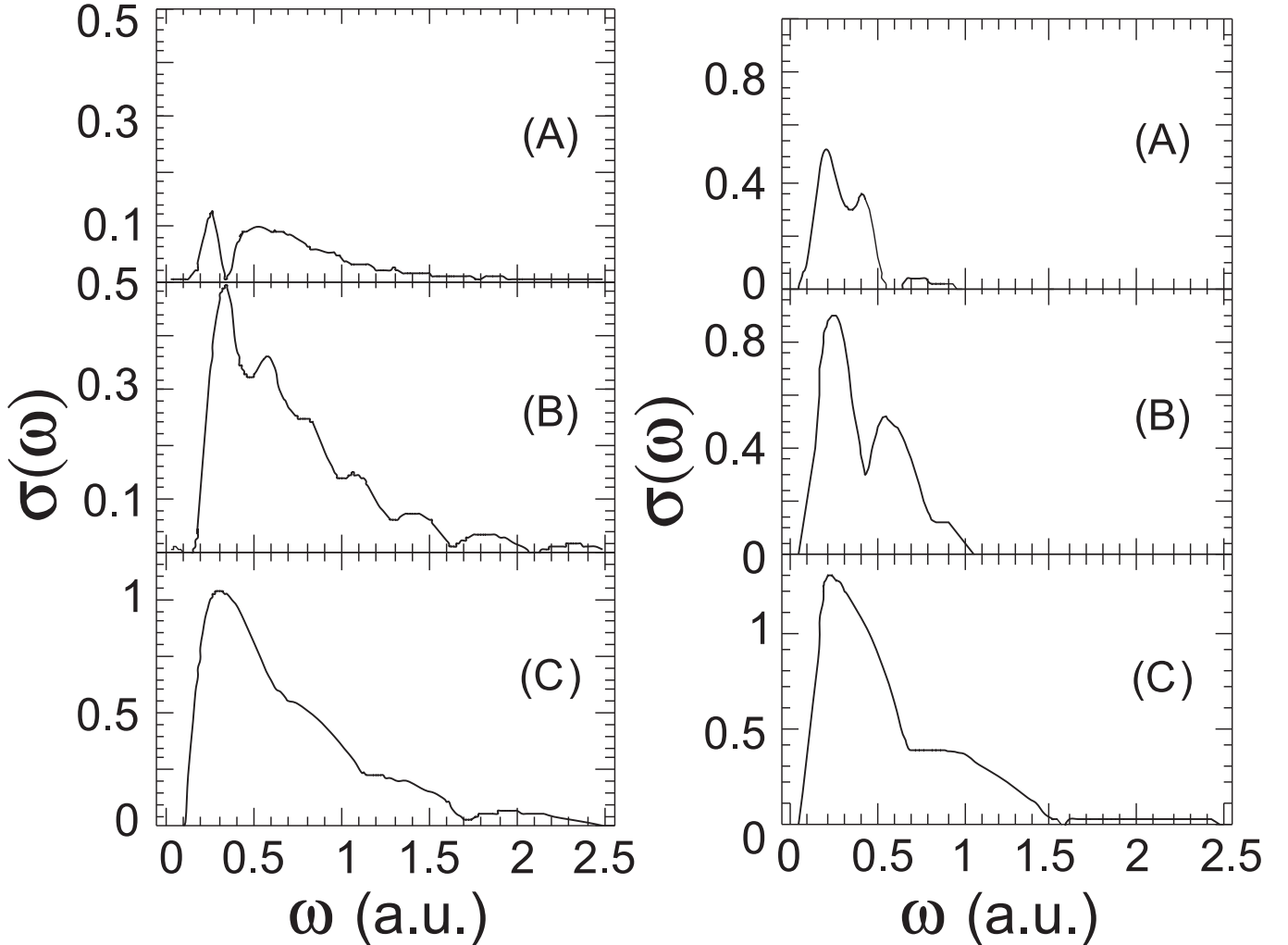
FIG. 4: Optical conductivity of Coulomb droplets at different parameters of the disorder. Left: short-range impurities. A − $\Lambda/\Lambda^* = 0.05$, B − $\Lambda/\Lambda^* = 0.2$, C − $\Lambda/\Lambda^* = 0.5$. Right: Coulomb impurities. A − $\Lambda/\Lambda^* = 0.1$, B − $\Lambda/\Lambda^* = 0.25$, C − $\Lambda/\Lambda^* = 0.5$.

that the spectrum broadens, loses its fine structure and its magnitude grows sizeably, in agreement with experiment. Overall, a qualitative description of all essential points in doping dependence of the optical conductivity of LSCO, reported in Ref.[11], becomes possible. Similarly, the curves at right panel of Fig.4 have been obtained for rings with $\alpha = 0.5$ given the following parameters: A − $\Lambda/\Lambda^* = 0.1$, $r_{out} = 36$, B − $\Lambda/\Lambda^* = 0.5$, $r_{out} = 15$, C − $\Lambda/\Lambda^* = 0.5$, $r_{out} = 7$. Again, the values of Λ in these examples meet the proportions: $\Lambda^A : \Lambda^B \simeq 0.15 : 0.07$ and $\Lambda^C : \Lambda^B \simeq 0.26 : 0.15$.

The experimental findings of Ref.[11] have been examined also within the diagrammatic theory of charged stripes[19]. In agreement with the CDW theory by Fukuyama and Lee[9], the authors have found the optical conductivity of an isolated stripe to display a peak at a pinning frequency ω_0, whereas the the spectra of an array of interacting stripes show a second peak at a

frequency ω_U, that depends on the interaction strength. Physically, the peak at ω_0 corresponds to the plasmon wing (that is not necessarily broad and may look like a single resonance, as at right panel of Fig.3), while the peak at ω_U has no direct counterpart in our model. However, the observed microwave spectra in cuprates generally show a much more complex structure with a number of resonances.

In conclusion, we have studied the optical response of the liquid of interacting 2D Bose droplets in presence of a random impurity potential – either Coulomb or short-range. Each droplet is a charged Bose particle, delocalized with a charge density $\rho(r)$ in a certain spatial region (disk or ring in the present work). As discussed in Sec.IIIA, the behaviour of this system is similar to that of CDW in strong pinning regime (a dilute distribution of strong pinning centers). The optical conductivity spectrum of the model consists of a broad asymmetric feature

(plasmon wing) with a rather complex oscillating shape. The oscillations are intimately related to the behaviour of the droplet structure factor $S = \rho(k)^* \rho(k)$. The main qualitative conclusion to be drawn from our analysis is that the optical response is not only related to the peculiarities of the energy level distribution, but may also reflect the details of the real-space inhomogeneity of the system, such as the charge density within the droplets of a (super)conducting phase. The model can possibly describe the microwave optical response of doped cuprates, where the signature of the collective mode excitations (density waves, stripes) are observed.

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